Q.P. Code – 22237

Second Semester B.Sc. Degree Examination, September 2020

(Semester Scheme)

MATHEMATICS

Paper II

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : Answer all the questions.

I. Answer any **FIFTEEN** of the following : **(15 × 2 = 30)**

1. Define singular and non singular matrices.

2. Find Rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 1 & 3 & 4 \end{bmatrix}$.

3. If X_1 and X_2 are two eigen vectors of the matrix, A corresponding to eigen value λ, then prove that $X_1 + X_2$ is also an eigen vector of A.

4. Define consistent and inconsistent system of linear equations.

5. State Cayley-Hamilton theorem.

6. Find the lengths of the polar subtangent and polar subnormal at the point $\theta = \frac{\pi}{6}$ on the curve $r = a\cos 2\theta$.

7. For the cardioid $r = a(1 - \cos\theta)$, show that $2ap^2 = r^3$.

8. Find the pedal equation for the curve $a^2 = r^2 \cos 2\theta$.

9. Find the radius of curvature for the curve $y = ae^{\frac{x}{a}}$.

10. Find the expression for the coordinates of centre of curvature.

11. Find the asymptotes parallel to the co-ordinate axes of the curve $x^2y^2 - y^2 = 2$.

12. Find the envelope of the family of the curves $y = mx + \frac{a}{m}$, where m is a parameter.

13. Define Node.

14. Find the area included between the parabola $y^2 = 4ax$ and its Latus rectum.

15. Write the expression for the volume of revolution of the curve $y = f(x)$.

16. Solve : $(e^y + 1)\cos x\,dx + e^y \sin x\,dy = 0$.

17. Find the integrating factor of $(1 + x^2)dy + (y - \tan^{-1} x)dx = 0$.

18. Solve : $p^2 - 5p - 6 = 0$.

19. Find the general solution of $xp^2 - yp + p^2 + 1 = 0$.

20. Define Orthogonal trajectories.

II. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. Find the rank of the matrix A by reducing into the normal form $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -3 \\ 3 & -3 & 1 \end{bmatrix}$.

2. Find the inverse of the matrix A by elementary transformations, where

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}.$$

3. Solve the system of equations :

$$\begin{aligned} x + 2y - z &= 3 \\ 3x - y + 2z &= 1 \\ 2x - 2y + 3z &= 2 \end{aligned}$$

4. Find eigen values and eigen vectors of $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$.

5. Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$. And find its inverse.

III. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. Find the angle between the curves $r = a$ and $r = 2a\cos\theta$.

2. Obtain the Pedal equation of the curve $r^m = a^m \cos m\theta$.

3. Find the radius of curvature of the curve $y = 4\sin x - \sin 2x$ at $x = \frac{\pi}{2}$.

4. Find the equation to evolute of the curve asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

IV. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. Find the envelope of the family of curves $x\cos^3\alpha + y\sin^3\alpha = a$ where α is a parameter.

2. Find all the asymptotes of the curve $x^3 + x^2y - xy^2 - y^3 - 3x - y - 1 = 0$.

3. Find the position and nature of the double points of the curve $x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$.

4. Trace the curve Folium of Descarte $x^3 + y^3 = 3axy$.

V. Answer any **TWO** of the following : **(2 × 5 = 10)**

1. Find the area included between the parabola $y^2 = 4x$ and the line $x + y = 3$.

2. The curve cardioid $r = a(1 + \cos\theta)$, revolves about the initial line. Find the surface area of the solid thus obtained.

3. Find the volume of the solid generated by revolving the curve asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the X-axis.

VI. Answer any **THREE** of the following : **(3 × 5 = 15)**

1. Solve : $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$.

2. Solve : $(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x$.

3. Test the exactness and solve, $(4x+3y+1)dx + (3x+2y+1)dy = 0$.

4. Find the general and singular solution of $(x^2-1)p^2 - 2xyp + (y^2-1) = 0$.

5. Find the orthogonal trajectories of the family $\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$, '$\lambda$' being a parameter.
